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Research on Innovative Method of Image Denoising Based on Micro Partial Equation

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ABSTRACT. With the continuous development of digital image processing technology, people pay more and more attention to the visual effect of images. We need and hope to get clear image quality, which requires perfect image denoising technology. The fractional partial differential equation difference method for image denoising is an important direction in the field of image denoising, and its numerical method research has important theoretical significance and practical value. This paper introduces two denoising models based on PDE, namely ROF model and LLT model. According to the comparison of these two models, a comprehensive model combining ROF model and LLT model with weight function is proposed. After analyzing the parallelism of solving the difference equation after discretization of the fourth-order PDE, the noisy image is divided into stripe overlapping data, and the image is denoised in parallel, which greatly reduces the running time.

KEYWORDS: Micro-partial equation, Image denoising, Global variational method

1. Introduction

In the process of digital image acquisition, transmission and storage, the image quality will be degraded due to various reasons, such as noise and blur in the image. In this case, image processing came into being. Digital image processing refers to the behavior that visual information in scientific research and production is digitally processed by computer, and through the processing of image information, people's visual psychology or application needs are satisfied. Pixel value can reflect the hierarchical details of an image, and it is one of the essential basic elements of an image. Another basic factor, the spatial resolution of image, can be expressed by the total number of pixels under the condition of resolution. In the literature [1], firstly, the ultrasonic image is denoised by curvelet transform, then the ultrasonic image is enhanced by multi-scale Retinex algorithm, and bilateral filtering is used instead of Gaussian filtering of Retinex algorithm to complete the ultrasonic image denoising. However, the above methods all have the problem of low stability. A fourth-order PDE model based on Laplace operator is proposed in the literature [2]. The fourth-order PDE denoising method can effectively remove noise, keep the edge and texture of the image, and eliminate the “ladder effect” produced by the second-order PDE method. It is a relatively successful image denoising method at present [3].

This paper studies how to introduce the geometric structure information into the existing second-order partial differential equation denoising model, and proposes a new denoising model, so as to get better denoising results. According to the characteristics of noise distribution, the model is improved, and an image denoising algorithm based on partial differential equation is proposed. The traditional partial differential denoising algorithm is integrated into hyperspectral images and acts on spectral curves in different bands, which makes good use of spatial information and spectral information, improves the denoising effect of hyperspectral images as a whole, and obtains ideal denoising effect.

2. Rof Second-Order Partial Differential Model

In theory, the image denoising technology based on partial differential equation is to construct a suitable partial differential equation according to the initial energy of the image to be denoised, and solve it according to the corresponding model, and the calculated stable solution is our ideal denoised image [4]. Energy functional optimization model based on variational principle and anisotropic diffusion model based on conduction coefficient of partial differential equation. Variational model reduces noise by optimizing the gray value of image pixels and constructing energy functional. It has the characteristics of multi-scale and can be denoised in different scale spaces, so the obtained picture keeps edges in smaller scale, has good denoising effect in larger scale, and the image is smoother.

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In 1992, Rudin L, Osher S and Fatemi E proposed a classical noise reduction method Rodin-Osher-Fatemi (ROT) model, whose basic idea is that the total variation of noisy images is always greater than that of noiseless images. Therefore, the ROF model can be expressed as the following optimization problems:

$$\min_u E(u), E(u) = \int_\Omega \left(|\nabla u| + \frac{\lambda_1}{2} \|u_0 - u\|^2 \right) d_x d_y \tag{1}$$

The optimization problem can be solved by transforming it into a partial differential equation by using the variational principle. The basic method is to find its Euler-Lagrange equation first, and then get the corresponding partial differential equation by using the gradient descending flow:

$$\frac{\partial u}{\partial t} - \nabla \cdot \left(\frac{\nabla u}{|\nabla u|} \right) = \lambda_1 (u_0 - u) \tag{2}$$

This is a second-order nonlinear diffusion equation, which can be discretized into an explicit difference equation:

$$u_{ij}^n = u_{ij}^{n-1} + \tau A_{ij}^{n-1} + \tau \lambda_1 \left[(u_0)_{ij} - u_{ij}^{n-1} \right] \tag{3}$$

Among them, $A = \nabla \cdot \left(\frac{\nabla u}{|\nabla u|} \right)$.

Experiments show that with the increase of iteration times, ROF model can remove noise well, but it is also found that the diffusion equation will have strong diffusion in the smooth region of the image. The existence, uniqueness, stability and convergence of the scheme solution are analyzed, and the accuracy analysis of the scheme solution is given. Finally, the Lena image with standard Gaussian white noise is denoised. This kind of model requires the energy functional of image resume and appropriate constraint conditions from the physical point of view, and obtains the corresponding Euler equation by variational method, and finally achieves the purpose of image denoising [5-6].

3. Fourth-Order Partial Differential Denoising Model

Although the use of second order is of great significance to image processing. However, the obtained image effect is sometimes unsatisfactory. For example, the image obtained by using the model for image denoising usually has a large jump in gray value, which makes the processed image show a block situation, and the visual effect is not ideal. The diffusion intensity is consistent in different directions, which leads to blurred edges while denoising the resulting image. If the iteration time is reduced and the edges are kept, the noise will not be completely removed. When the functional to be optimized is differentiable, the problem of image processing or computer vision can be solved by finding the stable solution of the partial differential equation derived from the functional. It is a lot of equations including the variables of time, and it also includes the variables of partial differential equations of mathematics and physics. We call these variables collectively, that is, it is called a new equation in evolution or a new equation in evolution.

Mixed order PDE image diffusion denoising based on nonsubsampled contourlet transform. The second-order PDE and the fourth-order PDE are used to process the image in high and low frequency bands, and the processed coefficients are transformed by nonsubsampled contourlet inverse transform [7], and the dimension is reduced by kernel principal component analysis algorithm. Under the joint action of the internal energy function and the external energy function, the elastic film will reach a certain balance, that is, to minimize the total energy, and the solution obtained in this balance state is the denoised image. We know little about noise information, because noise is random and diverse, unless we use statistical methods, which is not the goal of problem research. The image is understood as an elastic film made of abstract material deformed by external force. The physical characteristics of the film itself are described by internal energy function, while the external force exerted by the image on the film is described by external energy function.

The fourth-order PDE denoising method is based on an optimization objective functional, as shown below [8]:

$$E(u) = \int_\Omega f\left(|\nabla^2 u|\right) d_x d_y + \frac{\lambda}{2} \int_\Omega (u - u_0)^2 d_x d_y \tag{4}$$

In the formula, the first term is the regular term, which makes the result image tend to be smooth; The second term is fidelity term, which makes the result image consistent with the original image as much as possible, and keeps the image details. The parameter λ (> 0) is a Lagrangian constant, which depends on the noise level. $\nabla^2 u$ is Laplace operator. It is required that the function $f(s) \geq 0$ is an increasing function, and the smoothness of the image is measured by $s = \left|\nabla^2 u\right|$. Functional minimization is equivalent to smoothing the image, so that the denoising problem is transformed into a functional extremum problem. The fourth-order partial differential equation corresponding to formula (4) is obtained by using Euler equation principle and gradient descent method:

$$\frac{\partial u}{\partial t} = -\nabla^2\left[f'\left(\left|\nabla^2 u\right|\right)\frac{\nabla^2 u}{\left|\nabla^2 u\right|}\right] - \lambda(u_0 - u) \tag{5}$$

Let $c(s) = \frac{f'(s)}{s} = \frac{1}{1 + s^2 / k^2}$ be the diffusion control function, then equation (5) can be simplified as:

$$\frac{\partial u'}{\partial t} = -\nabla^2\left[c\left(\left|\nabla^2 u\right|\right)\nabla^2 u\right] - \lambda(u_0 - u) \tag{6}$$

Equation (6) is the current typical fourth-order PDE denoising model adopted in this paper.

4. Partial Differential Equation Denoising Model

4.1 Pm Model

By introducing the concept of physics into the image, the whole image can be regarded as a plate with different heat, and the heat equation changes with time, which makes the temperature of the whole plate tend to be balanced. Because Laplace operator has strong smoothness, it makes the image transition smooth, and even the edges and details are blurred. That is to say, an image is understood as an elastic film made of abstract materials, which is deformed by external force. When the same external force acts on the film surface, the final deformation result of the film depends on the physical properties (internal energy) of the material, which do not change with the change of the applied external force. A class of low-pass filters represented by Gaussian convolution is equivalent to solving the heat conduction equation with the original image as the initial value, which builds a bridge for the application of partial differential equations in image processing, and realizes strong smoothing in the smooth region of the image, while weak smoothing at the edge of the image to protect the edge.

The expression of PM model is as follows:

$$\frac{\partial u}{\partial t} = \nabla\left(g\left(\left|\nabla u\right|\right)\nabla u\right) \tag{7}$$

In formula (7), $g(s): R^+ \rightarrow R^+$ is a non-increasing function, and satisfies the following requirements: $g(0) = 1$, when $s \rightarrow 0$, $g(s) \rightarrow 0$. When the value of u is large, that is, when the gradient is at the edge of the image, we can draw the following conclusions from the definition of function $g(s)$: The diffusion speed can be reduced quickly at the image boundary, and then the clarity of the boundary can be maintained by related techniques; On the other hand, the value of u in the boundary region is very small, so the noise can be removed normally. Therefore, the diffusion of the equation in the image region is a selective condition by using the function gradient [9].

The gray scale information of the image is located in the low frequency band, which is relatively high frequency band, and the noise in the low frequency band is less. The second-order partial differential diffusion equation can not make the noise disappear in a large scale, but also lead to a high degree of gradient sudden change. In traditional image processing, image edge detection and image filtering are originally two independent processes, and this model combines the image edge detection process with the image denoising process to form a new mechanism. Then only the smooth area is de-noised, and the edge is not smoothed, which will make the boundary clearer. It can also be a weighted combination of these external forces. The criterion of its definition is to make the deformation model converge on the surface of the object, but not exceed this surface. Where the image gradient is small, the diffusion is large. In this way, the noise can be filtered and the gradient amplitude boundary can be kept to a certain extent. Maintain edges; The gradient is small, the diffusion coefficient is large, the diffusion intensity is large, and the image is blurred, thus realizing the anisotropic diffusion ability.

Let the initial gray image be $u(x,y,0)$, and $u(x,y,t)$ is a smooth image at time t. Then the heat conduction smoothing equation of the image is:

$$\frac{\partial u(x,y,t)}{\partial t} = \nabla u(x,y,t) \quad (8)$$

In the formula, $\nabla u(x,y,t)$ is the Laplace operator of the image, and its initial condition is $u(x,y,0)$. The solution of equation (8) is:

$$u(x,y,t) = G_t * u(x,y,0) \quad (9)$$

Here, $*$ represents convolution

$$G_t(x,y) = Ct^{-1}\exp\left[-\left(x^2+y^2\right)/4t\right] \quad (10)$$

Equation (10) is a Gaussian function. Therefore, the convolution of the initial image with Gaussian filters of different scales is equivalent to the solution of the heat conduction equation [10].

4.2 Adaptive Improved Variational Denoising Model

The traditional low-pass filter and the constant coefficient thermal diffusion equation mentioned above are used for noise reduction at the expense of fuzzy edges. To enhance the edge of blurred image, diffusion equation can be executed through high-pass filter or reverse on time axis. When the kernel principal component analysis algorithm is used for dimension reduction based on each subband, the reconstructed data obtained by kernel principal component analysis algorithm can reflect the amplitude characteristics of contour wave coefficients formed by noise, because most of the coefficients in each subband are contour wave coefficients formed by noise. Therefore, the update operation of image sub-block edge data needs the data of adjacent sub-blocks, so each process needs frequent communication in the calculation process, resulting in inefficient parallel algorithm. In order to improve the shortcomings of the above equation, the diffusion coefficient of this equation is not taken as the function of the gradient of the original noisy image, but determined according to the gradient of the image iterated in each step. The edge is estimated on the scale, and then the information is used to determine the degree of diffusion to avoid excessive diffusion at the edge points.

Because the model equation itself is ill-conditioned, it is easy to prove that the existence and uniqueness of the solution can be guaranteed only under strong conditions, so this initial value problem is ill-posed. Therefore, for a digital image, even if the scale of the fractional mask can reach the scale of the digital image itself, it is impossible to reach the analytical value of its fractional differential, and it can only approach this analytical value infinitely. To solve the variational problem of functional extremum is to solve the corresponding partial differential equation. According to the variational preparation theorem, we can transform this problem into solving the corresponding Euler equation. Its expression is actually equivalent to linear diffusion equation, but at the edge, its expression is different. In fact, the noise on the edge has not been effectively suppressed in this process. When the boundary of a sub-image is not the boundary of the whole image, a certain pixel line is extended up and down, and a division with overlapping regions is obtained. By properly selecting the conduction coefficient, the anisotropic diffusion can be filtered and enhanced at the same time, and it goes forward on the time axis. The stability of diffusion can be guaranteed by the extremum principle.

The global variational method (TV) model can not only remove noise but also maintain edges. Next, the classical algorithm and the diffusion property of the model are introduced in the case of isotropic TV norm. In fact, the time parameter t is introduced. From the idea of gradient descent, we can get the diffusion equation of image u evolution with time.

$$\frac{\partial u}{\partial t} = div\left(\frac{\nabla u}{(|\nabla u|)}\right) - \lambda(u-u_0) \quad (11)$$

In the image diffusion equation, the local coordinate system η - ξ is a powerful tool to analyze the diffusion performance of the equation. As shown in fig. 1. The θ axis is used to represent the gradient direction parallel to the image at a certain pixel point, and the ξ axis is its corresponding vertical direction.

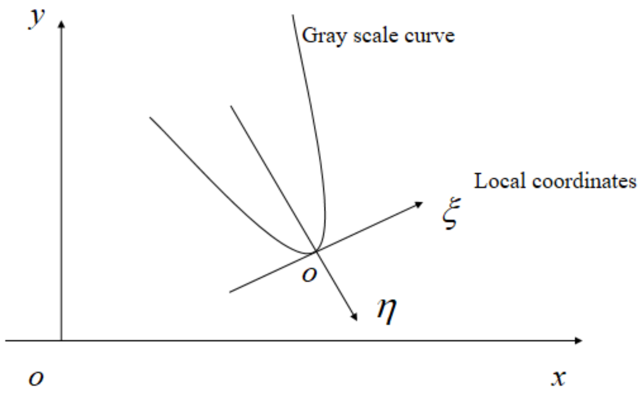

Fig.1 Schematic Diagram of Global Coordinates and Local Coordinates

Considering the adaptive TV model and adaptive LLT denoising method, the diffusion coefficient function $\alpha(x)$ is introduced into the above model; Get the following functional:

$$\min_u \left(\int_\Omega \alpha \cdot \sqrt{(u_x + u_{xx})^2 + (u_y + u_{yy})^2} + \lambda \int_\Omega (u_0 - u)^2 \right) \quad (12)$$

Among them, $\alpha = \dfrac{1}{1 + k\left|G_\sigma * \nabla u\right|^2}$ has the same function as the diffusion coefficient function in adaptive TV and adaptive LLT models, which adaptively adjusts the smoothness, performs weak diffusion processing at the image edge and strong diffusion processing in the flat homogeneous region.

4.3 Anisotropic Diffusion Filtering Method Based on Divergence

It is difficult to get a good analytical form for the definite solution of most partial differential equations, and numerical methods provide a way to solve such problems. A main process reads in the image to be processed, and distributes several sub-images to each sub-process according to the above-mentioned data partition mode, so that diffusion is carried out along the edge direction at the edge instead of along the direction perpendicular to the edge, which is called anisotropic diffusion equation.After the second-order partial differential equation denoising model is used to process the image, sometimes "blocky effect" appears, that is, the processed image has the same gray area. This second-order partial differential equation will evolve to the plane image when the support of the image is infinite, and finally be fixed to the plane image. For images with limited support, in order to avoid boundary distortion, symmetric boundary condition is usually used, that is, the image takes the same value symmetrically on both sides of its boundary. Although tensor diffusion can be expressed as the sum of two orthogonal one-dimensional diffusion, its diffusion coefficients are different, so this kind of diffusion is called "anisotropic diffusion" [11], which is fundamentally different from isotropic diffusion in nature.

Coherent enhanced diffusion mainly deals with one-dimensional structural features in images. The eigenvector D of diffusion tensor constructed by coherent enhanced diffusion is the same as that of structural tensor J_ρ. The structure tensor is expressed as follows

$$J_\rho(\nabla u_\sigma) = K_\rho * \left(\nabla u_\sigma \nabla u_\sigma^T\right) \qquad (\rho \geq 0) \quad (13)$$

The direction of the eigenvector corresponding to the smaller eigenvalue of J_ρ is called coherence direction. The eigenvalue of D is taken as

$$\lambda_1 = \alpha$$
$$\lambda_2 = \begin{cases} \alpha & if\ \mu_1 = \mu_2 \\ \alpha + (1-\alpha)\exp\left(-\dfrac{C}{(\mu_1-\mu_2)^2}\right) & else \end{cases} \tag{14}$$

Among them, μ_1, μ_2 are the eigenvalues of $\nabla u_0 \otimes \nabla u_\sigma = \nabla u_\sigma \nabla u_\sigma^T$, and $C > 0, \alpha \in (0,1)$. When $\mu_1 - \mu_1 > C$, $\lambda_2 \approx 1$, when $\mu_1 - \mu_1 < C$, $\lambda_2 \approx \alpha$.

Gray images are linearly diffused. The longer the diffusion time is, the more blurred the image is. Finally, when the gray scale of the image spreads to the average value of the image, the diffusion stops. Although the “blocky effect” of second-order nonlinear diffusion equation denoising is eliminated, the denoised image is more satisfactory. The noise at the linear structure such as edge texture is effectively suppressed, and the blurring at the edge is reduced. However, it evolves slowly where the gradient is large. Therefore, after a period of evolution, the image will look like it is composed of horizontal areas with different intensity values. The boundaries of these different areas may be just boundaries, but they may also be in large smooth areas or inclined areas.

5. Simulation Experiment Analysis

This section verifies the denoising ability of the new model through experiments. The method of reference [12] was selected in the experiment. There are three main ways to evaluate the denoising results: from the image of denoising results. That is, the denoising result of the model is visually superior to other methods; From the residual image between the denoised result image and the original noiseless image. If there is less texture information left in the residual image, the method is considered to be excellent. From the quantitative index of denoising results. In this section, the signal-to-noise ratio is selected to evaluate the denoising results of each model. The larger the signal-to-noise ratio, the better the denoising method. For images of the same size, the execution time decreases almost proportionally with the increase of processor nodes. Meanwhile, with the increase of image size, the reduction of parallel algorithm execution time also increases. The simulation experiment flow is shown in Figure 2.

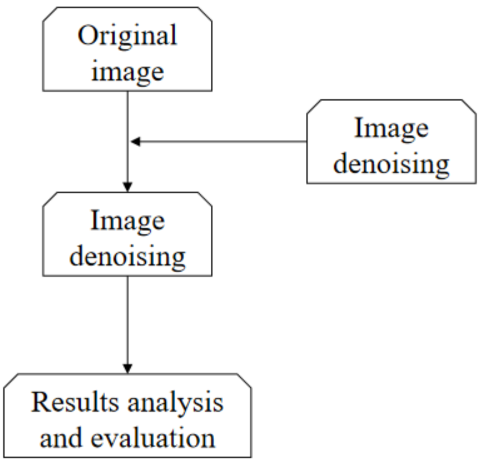

Fig.2 Flow Chart of Simulation Experiment

In this experiment, the noise image obtained by adding Gaussian white noise with mean value of 0 and variance of 0.03 to lena image is used as experimental material, and is programmed in matlab2011. The experiment has gone through 150 iterations. In particular, the renderings of the 50th iteration, the 100th iteration and the 150th iteration are displayed, as shown in Figure 3.

Fig.3 Operation Result of Pm Algorithm

After calculation, the SNR values of these three iterations are listed in Table 1.

Table 1 de-Noising Effect of Gaussian Filtering Algorithm

Iteration times	50	100	150
SNR	20.2517	22.5877	21.6933

It can be seen from the SNR data in the table that the value of the 100th iteration is larger than that of the 150th iteration, which shows that the overall denoising effect of the 100th iteration is better than that of the 150th iteration, because the higher the iteration times, the less obvious the boundary of the image is.

To test the denoising performance of this algorithm, the hard threshold algorithm, soft threshold algorithm and this algorithm are used to denoise the image in the contourlet transform domain, and the results are shown in Table 2.

Table 2 Image Denoising Effect of Three Noise Intensities under Different Algorithms

Noise intensity/dB	Hard threshold algorithm	Soft threshold algorithm	This algorithm
30	22.31	29.27	29.83
40	25.01	27.88	32.43
50	24.83	30.14	36.74

It can be seen from Table 2 that the hard threshold denoising algorithm has more noise residue and poor denoising performance; Soft threshold denoising algorithm has poor image detail preservation and denoising performance; However, the algorithm in this paper can not only remove noise effectively, but also preserve image details well, and has good denoising performance.

6. Conclusion

In this paper, the numerical methods of image denoising for spatial fractional partial differential equations are mainly discussed, and two difference schemes are constructed for the partial differential equations equivalent to the spatial fractional TV model. Secondly, the multi-scale analysis algorithm based on contourlet transform is used to locally denoise the image. The contourlet transform method is used to perform scale analysis and direction analysis on the denoised whole image at different times, and the whole image is decomposed in different directions at different scales to obtain different sub-bands. Compared with other models, the ladder effect of partial differential equation denoising model has the highest parameters such as peak signal-to-noise ratio, and at the same time, its operation efficiency is better than other models. By improving the existing denoising model, a better image denoising technology is proposed, and a new denoising model is proposed for hyperspectral images. Through a series of experiments, the results show that the proposed algorithm is effective and has better denoising effect than the traditional model. Partial differential equation denoising algorithm can quickly deal with the problem of image denoising, greatly improve the efficiency of problem processing, and has good application value and development prospects. It is widely used in various fields of image processing, such as image denoising, image segmentation, target tracking and so on.

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